22103175

QP CODE: 22103175

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE EXAMINATIONS, OCTOBER 2022

Second Semester

Core Course - MM2CRT01 - MATHEMATICS - ANALYTIC GEOMETRY, TRIGONOMETRY AND DIFFERENTIAL CALCULUS

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2017 ADMISSION ONWARDS

8CC46AE4

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Derive the condition that the line y = mx + c is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
2. Derive the equation of chord of contact of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
3. *Chords of a parabola are drawn through a fixed point. Show that the locus of their middle points is another parabola.*
4. Find the condition that the lines lx + my + n = 0 and $l_1x + m_1y + n_1 = 0$ to be conjugate with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
5. Derive the polar equation of a parabola.
6. Find the equation for a circle centered at the pole. Give an example.
7. Prove that $\sin 3x = 3 \sin x - 4 \sin^3 x$.
8. Prove that $\tanh^2 x + \operatorname{sech}^2 x = 1$.
9. Factorize $x^9 + 1$
10. If $y = e^{-x}(Ax + B)$, prove that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$.
11. Find the n^{th} derivative of cos(ax+b).

12. Determine $lim[\frac{1}{x-2} - \frac{1}{log(x-1)}]$ as $x \to 2$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Two tangents from a point to the parabola $y^2 = 4ax$ make with each other an angle 45^0. Prove that the locus of their point of intersection is given by $y^2 - 4ax = (x+a)^2$.

14. Find the orthoptic locus of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

15. Find the equation of the polar of (x_1, y_1) with respect to (a) the parabola $y^2 = 4ax$ (b) the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

16. A tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, whose centre is C, meets the circle $x^2 + y^2 = a^2 + b^2$ at Q and Q_1. Prove that the lines CQ and CQ_1 are conjugate diameters of the ellipse.

17. Replace the polar equation $r^2 = 4r\cos\theta$ by equivalent cartesian equation, and identify its graph.

18. Sum the series $\frac{1}{2}sin\alpha + \frac{1.3}{2.4}sin2\alpha + \frac{1.3.5}{2.4.6}sin3\alpha + \ldots$

19. Sum the series $sinh\alpha - \frac{1}{2}sinh2\alpha + \frac{1}{3}sinh3\alpha - \ldots$

20. Find the n^{th} derivative of $y = \frac{x}{x^2+a^2}$.

21. Determine $lim[2 - \frac{x}{a}]^{tan\frac{\pi x}{2a}}$ as $x \to a$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. If P and D are the extremities of semi-conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, show that

(a) the locus of the middle point PD is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2}$.

(b) the locus of the point of intersection of the tangents at P and D is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$.

(c) the locus of the foot of the perpendicular on PD from the centre of the ellipse is $a^2x^2 + b^2y^2 = 2(x^2 + y^2)^2$.

23. A chord PQ of a conic subtends an angle of 2β of constant magnitude at the pole. Find the locus of the intersection of the tangents at P and Q.

24. Separate into real and imaginary parts $sin^{-1}(cos\theta + isin\theta)$, where θ is real.

25. (a) If $y = sin(msin^{-1}x)$, show that
$(1-x^2)y_{n+2} = (2n+1)xy_{n+1} + (n^2 - m^2)y_n$ and find $y_n(0)$.
(b) If $f(x) = x^2 tanx$, prove that $f^n(0) - {}^nC_2 f^{n-2}(0) + {}^nC_4 f^{n-4}(0) - \ldots\ldots = sin\frac{n\pi}{2}$.

(2×15=30)